

# JOURNAL OF DoD RESEARCH & ENGINEERING



## Reliability Assessment of a Single-Shot System by Use of Screen Test Results

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## About

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The JDR&E, sponsored by the Under Secretary of Defense for Research & Engineering and managed by the Defense Technical Information Center, publishes innovative articles that have the highest current and potential impacts on the warfighter's mission. The JDR&E is a semiannual, peer-reviewed journal of classified and controlled unclassified scientific and technical research conducted across DoD laboratories and warfare centers. We welcome submissions from all areas and will protect all sensitive information. For more information on article submissions and the peer-review process, please visit <https://go.usa.gov/xnsx8>.

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## Greetings from Ms. Mary J. Miller, Principal Deputy Assistant Secretary of Defense for Research and Engineering

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Our near-peer adversaries' investments in research and development have increased dramatically, as a result, the United States is no longer able to boast of technology superiority. Our adversaries watched us for years and saw our equipment, studied our tactics, techniques and procedures, and determined our Concepts of Operation. They are investing in technology and capabilities attempting to avoid our strengths and exploit our weaknesses. To maintain our technological edge, our scientist and researchers must work together to develop new and advanced capabilities and technologies.

Technology changes at great speed; therefore, we must continue to push the limits and work with our stakeholders – academia, industry, Congress, and our allies. We face the competing necessities of protecting our sensitive research and collaborating with partners outside of the DoD. Our scientists, researchers, and engineers must continue to have the ability and opportunity to gain peer-review status for limited or classified projects, and to share information and further leverage discoveries.

In an effort to ensure an alternative peer-review framework exists for the research and engineering enterprise, we initiated a peer-reviewed journal forum to afford our scientists and technologist an opportunity to share their research in a controlled access environment. The Defense Technical Information Center (DTIC) developed an internal journal, Journal of DoD Research and Engineering (JDR&E), to recognize the science and technology community's best works at the Controlled Unclassified/Classified level.

Through its print and online incarnations, JDR&E aims to publish influential papers to significantly advance scientific understanding. The JDR&E selections present novel and broadly important data, syntheses, or concepts. They should merit recognition by the wider DoD science and technology community and address the need to collaborate, recognizing contributions from our research community, while reducing the information available to the public and the potential for exfiltration.

The JDR&E welcomes submissions from all fields of DoD S&T. The editors are committed to promptly evaluating and publicizing submissions while upholding high standards that support reproducibility of published research. The JDR&E will be published semiannually and selected controlled unclassified publications will be available in print, while classified publications will be available online.

The JDR&E is a critical piece of the U.S. retaining technological superiority over our adversaries. We must all work together in order to stay ahead and promote the exchange of data and concepts vital to the Department of Defense in the years ahead.

## Letter to Our Readers from the Principal Director, Research, in the Office of the Assistant Secretary of Defense (Research and Engineering), Mr. Dale A. Ormond

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I am delighted to welcome you to the Journal of DoD Research and Engineering (JDR&E). This new addition to our family of printed and digital peer-reviewed publications will foster and facilitate our collaboration across the S&T Enterprise, including the 17 Communities of Interest (CoI). Like many worthy endeavors, the journal's foundation is rooted in a simple goal: to encourage our researchers to publish their classified and controlled unclassified (C/CU) works and to ensure that the experts on our CoIs who selflessly volunteer to serve as reviewers, validate their works.

As the Principal Director for Research, I have oversight of all of the DoD's S&T investments, and lead the Reliance 21 planning effort. Most importantly, my job is to ensure that the long-term strategic direction of the Department's S&T programs will continue to generate extraordinary innovations to maintain our Military's operational and technological superiority to support our warfighter. My job depends on the expertise of people like you, the researchers and engineers who work in our DoD Labs and serve in our research communities. For a long time, you have shouldered tremendous responsibility on behalf of the warfighter and continued to take on the task of keeping them safe. However, due to the sensitivity of your work, you have not been able to gain the recognition that you deserve for your efforts. I hope to inspire you to take advantage of this unique opportunity to submit your exciting work to this journal. We want to give you the chance to highlight the support you have received from your organization over the years and recognize you for your accomplishments. The Journal of DoD Research and Engineering is here to provide a secure venue for you to publish your work, to facilitate your connection to the larger DoD R&E communities, to promote your collaboration in cross-cutting research areas, and to enable you to share and use the Defense Technical Information Center (DTIC) repository of linkable scientific and technical information. To all the DoD Laboratory and Warfare Center Directors who are making good use of the Section 219 Authorities authorized in the Duncan Hunter National Defense Authorization Act for Fiscal Year 2009, you now have a venue by which you can meet the publishing requirements.

From the silent distance of space where your robots roam and service our satellites, as Dr. Roesler expressed so eloquently in his op-ed article, to the solitude of your lab where your research efforts begin, or wherever you are, I encourage you to publish, volunteer, and/or serve as the journal's reviewers. Reach out to the larger communities of R&E to collaborate and coordinate. Our warfighters depend on you to continue to climb to the summit of your field and do the impossible. I wish you all great success with your important research as you work, publish, and help us ensure the S&T we continue to pursue is well beyond any other nation on Earth.

## Letter from the Administrator of Defense Technical Information Center, Mr. Christopher E. Thomas



I am excited to welcome you to the inaugural issue of the Journal of DoD Research and Engineering (JDR&E). During my tenure at the Defense Technical Information Center (DTIC), I have noted the need for a venue that provides our researchers and scientists the capability to share their work, while still affording the protections needed to maintain the competitive advantage our S&T community provides us. The JDR&E is a peer-reviewed journal of DoD classified and controlled unclassified (C/CU) articles that cover the scientific and technical research conducted within the Department. Under the guidance of Ms. Mary Miller, Principal Deputy Assistant Secretary of Defense for Research and Engineering, and the management by DTIC, the JDR&E will serve as a unique platform to encourage collaboration and coordination between all 17 Communities of Interest, from Advanced Electronics to Biomedical to Space, to meet the DoD's S&T challenges of today and tomorrow. The JDR&E's key functions are to:

- Recognize researchers, scientist, and engineers doing exceptional work at the C/CU level and provide a secure platform to share their research
- Increase DTIC repository capability to receive, preserve, manage, and disseminate scientific and technical knowledge generated from the best DoD S&T investments
- Promote multi-agency collaboration in cross-cutting R&E and serve as a pipeline for DoD future breakthroughs

DTIC will manage the JDR&E's semiannual publication to ensure all C/CU R&E articles are validated via a peer-review process. The JDR&E is a venue for researchers across DoD to publish their sensitive work and avoid information exfiltration, a possible risk often associated with the publication of data in academic journals. DTIC's security protocols will protect against this threat—ensuring the right balance between the need to share S&T activities while protecting this body of work from our adversaries. We have many excellent researchers in our DoD Labs, and this journal provides a structured, secure, and premier venue to publish their work, recognize their talent, and highlight their extraordinary accomplishments and discoveries. To be successful this journal needs the entire community to participate by submitting articles, serving on the editorial board, and volunteering as peer reviewers. My vision for the JDR&E is straightforward: to protect, advocate, and serve as a gatekeeper of C/CU generated by DoD's best S&T investments. It will help catalyze new discoveries and world-leading innovation to provide our warfighters with the technologies that will assist the Department in maintaining its technological advantage.

I would like to take this opportunity to thank Ms. Mary Miller. Without her vision, passion, and commitment, this journal would not exist. I owe a tremendous debt of gratitude to her for her guidance and support during the publication of the journal's inaugural issue. I am grateful for the support of Mr. Dale Ormond whose tremendous effort enabled us to connect with the CoI and DoD laboratory community. I am in debt to Dr. Gordon Roesler from DARPA and the authors from ARDEC for graciously agreeing to contribute to the journal. I appreciate the assistance provided by my staff, Mr. Roger Garay, Ms. Michele Finley, Mr. Brent Ishizaki, and Mr. Jason Lawrence. I also wish to thank the editorial and production staff for their hard work, professional acumen, and devotion to the journal. Finally, many thanks to you, our readers, who join us on this journey! Your insights and feedback will help to shape this journal into an effort that benefits the warfighter - whom we all support.

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## Journal Contributions

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The Journal of DoD Research & Engineering (JDR&E) would like to extend our sincere gratitude to the authors from U.S. Army Armament Research, Development and Engineering Center (ARDEC) whose contributions are essential for the publication of our journal's inaugural issue. We also owe a tremendous debt of gratitude to ARDEC senior scientist, Dr. Don Carlucci, for his assistance and unwavering support of our mission.

We deeply appreciate the contributors from Naval Undersea Warfare Center (NAVSEA) for their swift response to our call for submission, and in so doing, significantly improved the breadth and depth of our introductory issue.

Finally, we would like to take this opportunity to thank Dr. Gordon Roesler from DARPA, who on short notice, graciously agreed to contribute the journal's first op-ed article, which we found both informative and delightful to read.



## Reliability Assessment of a Single-Shot System by Use of Screen Test Results

**Abstract:** Field reliability prediction methods based upon early screening results typically involve tracking a temporal metric such as on-time across a constant stress testing regime in order to model wear-out. These methods have very limited applicability to single-shot systems because reliability is not driven by wear-out, and testing is often performed at varying stress levels. A new methodology is introduced to track and project the reliability for a single-shot system as it goes through a multi-staged screening process that produces no meaningful temporal metric and involves significant differences in test strength. The approach described here assumes that the defect density during testing takes the form of an exponential decay, although other mathematical functions can be substituted for the exponential. In order to apply the decay rate function to a discrete pass/fail test scheme, the approach provides for normalization of the disparate tests to constant stress by back-calculating and adjusting for test strength based upon previous screening results. This approach is most useful when reliability does not involve wear-out of parts, which is typically true of single-shot systems. However, it also potentially has utility for all programs that need to glean information about early failures caused by fabrication problems, so long as a discrete end point for the reliability requirement such as warranty termination has been established. The equations provide a tool with which reliability practitioners can estimate field reliability of a new lot of single-shot or warrantied systems based upon early screen results, as long as a complete set of data from previous lot testing is available. Utility for reliability growth estimation is also described. A numerical example is given to demonstrate application of the model. This paper was adapted from a limited-distribution technical report by the same authors (Coate and Skaggs 2016).

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### 1. Introduction

Modern single-shot systems such as guided munitions rely on complex electronic guidance and control systems. Generally, part wear-out can be ignored in the service life requirements of the electronics in single-shot systems. This means that reliability is driven exclusively by what have been termed “infant mortality” or “extrinsic” defects such as bad parts or poor workmanship.

Elimination of infant mortality defects in single-shot systems typically involves stress screening at levels that precipitate failures without substantially reducing the life of the system. Environmental Stress Screening (ESS) is a process or series of processes in which environmental stimuli, such as rapid thermal cycling and random vibration, are applied to electronic items in order to precipitate failure of latent defects (Defense 1993). The objective is to remove as many of the significant defects that tend to dominate the reliability of fielded products during early life. The resulting product should then more closely exhibit the inherent or designed-in reliability (Bierbaum 2010).

There are quantitative methods to measure the effectiveness of an ESS program, such as those identified in (Defense 1993). These methods often give the program a sense of the initial number of patent and latent defects prior to screening and then the estimated latent defects that escape to the field. Using these results, the program can get a good estimate of the field reliability of the product. Because of the physics of latent defect distribution in a hardware system, the failures collected as a function of the amount of time the screen is applied resembles an exponential decay curve similar to the front end of the well-known “bathtub curve” (Modarres, Kaminskiy and Krivosv 2010). After an ESS program has matured, the users will have the parameters to create the failure rate exponential decay curve, which allows them to make a reasonable estimate of the field reliability based solely on the number of patent defects precipitated during first pass testing (Defense 1993).

During early tests of electronic systems, the circuit cards, interconnects, and other components that make up the subsystem get functionally tested in a binomial success/fail manner. When a failure occurs during one of

these tests, fault isolation is performed and the responsible component/assembly/CCA is fixed or replaced so that the subsystem can move onto the next test phase. For single-shot systems, the total test duration bears no relationship to actual usage of the system and so any specified temporal metric (on-time/MTBF/etc.) is not recorded or tracked. Due to this, program-level decisions are usually based upon the reliability metric of mission success rate at the system level rather than a temporal metric such as MTBF at the subsystem level. Mission success rate is easily defined and measured during system level testing (Sherwin Sep 2009). However, by the time that those tests are performed in any significant quantity, it is generally too late for corrective actions such as design or process changes to significantly improve reliability. This is why there is a clear need for a reliability tool that uses binomial success rate starting with first pass test results to provide pertinent and useful data to assist the decision-making process.

## 1.1 Model Approach

A model that tracks latent defect removal rate can show where the hardware is on the “bathtub curve” (actually failure rate decay curve) when it enters service. If the failure rate has flattened out due to removal of infant mortality defects, this provides confidence that the field reliability approaches the inherent reliability. Similar approaches have been developed to track the efficacy of reliability improvement efforts. It should be noted here that defect decay rate, as modeled in this paper, is conceptually distinct from reliability growth (the former develops a curve, the latter looks to reduce the area under/over the curve).

A survey of discrete reliability growth models is presented in (Fries and Sen 1996), which consists of a compilation of model descriptions, characterizations, and insights. The method of refining estimates of defect density and screening strength (Procedure D) plots the failures collected as a function of the amount of time the screen is applied. The resultant plot is that of an exponential decay curve. The main difference between the MIL-HDBK-344A and the discrete models is that one is continuous and time based, while the other consists of discrete test stages. The MIL-HDBK 344A procedure D method fits with the aim and expectations of these complex single-shot test programs (latent defect removal rate as a function of test step). However, there are major differences, since procedure D stipulates: 1), continuous time based versus discrete test stages, 2), one configuration level, and 3), constant stress.

The model developed in this paper will take discrete data during sequential tests at different stresses and configurations levels as an input and normalize the data so that the methods similar to those used in the MIL-HDBK-344A can be used to track and project the reliability achieved from screening of a single-shot system. The model provides a way to quantify both the relative strength or effectiveness of each test stage, and the effectiveness of the entire screen. The parameters of the exponential equation produced by the model can be used to project field reliability more accurately. The model will also allow for early test results to be used more effectively as feedback for process or design improvements. And lastly, the decay curve can be used for showing reliability growth by reduction of the area under the curve.

The approach is described in Section II, which includes: 1) methodology; 2) a discussion of the data required; 3) a list of model assumptions; 4) an expression for determining test equivalency; 5) an expression for system failure counts at test stage; 6) a decay rate parameter estimation procedure based on current lot data; and 6) a fielded reliability projection. Section III shows an example calculation for a complete data set, which includes an estimation of all parameters in the model and a projection of the field reliability. Section IV discusses application of the model. Concluding remarks and areas for future work are given in Section V.

## 2. Method

The method used in this approach can be described as follows:

1. Collect a complete pass/fail data set from a prior application of the screening program.
2. Apportion the discrete test steps into a scheme that can be normalized such that tests at different configurations levels can be used to create a single decay curve.
3. Back-calculate the Test Effectiveness Factor (TEF) of each test from the full data set by determining the fraction of remaining defects precipitated by that test.
4. Use linear regression to estimate the key screen parameters  $\alpha$  and  $\beta$  from the graphed data, which represent defect density of the tested population, and the overall strength of the screen, respectively.

5. Use the  $\alpha$  parameter from the next lot of test data to project field reliability.
6. With TEF and  $\beta$  fixed by the test program, measure reliability improvement from lot to lot by tracking reductions in  $\alpha$ .
7. Monitor reliability growth by looking for improvements in the screen parameters over time to detect improvement in the projected field reliability.

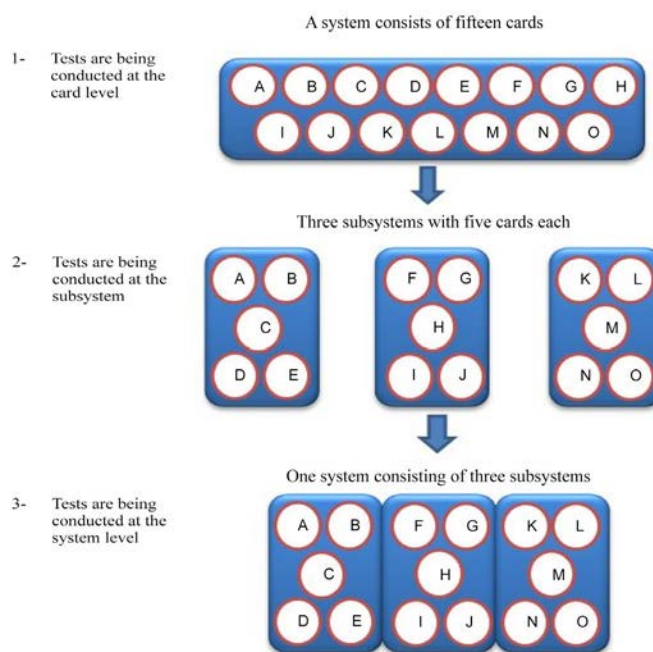
## 2.1 Data Required

The projection model developed here requires a specific type of data. The data that will be normalized to system level consists of a complete set of binomial test results at each of the independent test stages (ie. first past yield results at the card level up to system level field reliability and all tests in between). Though the individual test stages  $i=0,1,\dots,m$  can be performed at different configuration levels (card level, sub-system, system) the use of apportionment allows for lower level tests to be rolled up into system level results.

The methodology can be applied to any system. As an example, suppose a system consists of fifteen cards divided into three subsystems with five cards each (see figure 1). The test program arbitrarily declares fifteen cards, one each of each type by serial number, as being a “system equivalent” for reliability tracking purposes. At this point, there is no obligation to build the system from those particular fifteen cards. The fifteen cards are tested at the card level, and the number of functional failures is scored as the initial data point for the reliability model (Test Stage  $i=0$ ). If three functional failures occur on three separate cards from the population of fifteen, this is scored as three system failures for one system level test. If there were 4 other sets of 15 cards tested and no other failures occurred, the total score for that test stage would be 3 system failures out of 5 total system level tests. The assumption made is that the overall system level results at a particular test stage will be of binomial form. So although there can be multiple failures per system, the overall distribution follows the binomial form where the number of failures in  $T_i$  trials is  $N_i \sim \text{Binomial}(T_i, p_i)$  and  $0 < p_i$ .

After scoring, the three cards are repaired and returned to the population. At this point the test program may choose to perform a second card-level test such as a HASS test (Test Stage  $i=1$ ). This would be easily accommodated using the

same card-level equivalency. Next the cards are built into subsystems, although the fifteen cards may not be used in the same subsystem. The test program now arbitrarily declares one each from the three different subsystems as a “system equivalent” and tracking is no longer at the card level for reliability purposes but rather at the subsystem level. The subsystems are tested and then built into actual systems and equivalency is no longer necessary. The example within this paper will make this method of apportionment clear.



**Figure 1** – Scheme for system equivalency across different testing levels.

It should be noted that the apportionment method described here necessarily introduces some infidelities. One example is that failures can occur in a chassis that contains interconnects, but those interconnects are not tested at the card level phase. Another infidelity associated with testing under environmental stimulation (temperature, vibration, etc.) is that some failures will be precipitated during one test phase but not detected until a later test phase. The effect of these infidelities is largely mitigated by the use of test effectiveness factors as described later in this paper.

The main points are that (1) any test infidelities are of discrete magnitude and become less significant as the sample size grows, and (2) the infidelities remain consistent throughout the program and so do not impact the use of the model to show reliability improvement or deterioration.

Through the use of apportionment, the data will consist of  $T_i$  and  $N_i$ . The numbers of trials  $T_i$ , at test stage  $i$  and the count data  $N_i$  for the observed failures at that test stage are obtained directly from testing. Once the parameters of the model have been obtained for a test program, the test program must remain constant for future projections. Any changes in any of the tests or the sequence of the tests would require a new calculation of the model parameters.

## 2.2 Determination of Test Equivalency

The proposed model allows a program to take results from tests that occur at different stress levels and different system configuration levels, combine them and determine an overall screen effectiveness, and project system reliability post screen. Since the assumed exponential form results from testing under constant stress, the individual test stages that make up the screen must be normalized to constant stress. This is accomplished by calculating the relative effectiveness of each test against the entire screen. This relative effectiveness will be referred to as Test Effectiveness Factor (TEF). The TEF of an individual test stage is given by

$$d_i = \frac{\lambda_{i,1}}{\lambda_{i,2}} \quad (1)$$

where,

$$\lambda_{i,1} = \frac{N_i}{T_i} \quad (2)$$

$$\lambda_{i,2} = \sum_{j=i}^{m+1} \frac{N_j}{T_j} \quad (3)$$

$\lambda_{i,1}$  is the total number of failures during the test stage, and  $\lambda_{i,2}$  is the total number of failures that will occur through the entire test program beginning with the current test stage.

The expression for TEF found in Eq.1 is similar to an expression developed by the Army Materiel Systems Analysis Activity (AMSAA) for Fix Effectiveness Factor (FEF) during the Test-Analyze-and-Fix (TAAF) process (PM, WJ and WJ 2000). The FEF expression is used to calculate the effectiveness of an individual fix as a remedy to a particular failure mode. In the model described in this

paper, the TEF expression found in Eq.1 is used to assess the effectiveness of a test phase at reducing the latent defect density of a population. Each test phase within the screen reduces the fraction of latent defects within the population by a different amount and the magnitude of the fraction reduction is directly related to the strength of the individual test. The strength of a test phase is obtained from the ability to both precipitate failures and then detect those failures. This typically results in a HASS test having a higher TEF than that of a bench level functional test. For additional information on the application of the TEF see the example calculation in section III.

## 2.3 Model Assumptions

1. Test trials at the card and subsystem level have the potential for one or more failures at the system level such that total failures per system test can be larger than 1.
2. Each test trial at the system level results in a dichotomous success/failure outcome such that  $N_{i,j} \sim \text{Bernoulli}(p_i)$  for each  $i=0,1,\dots,m$  and  $j=1,\dots,T$ .
3. The distribution for the number of failures in  $T$  trials at system level test stages is binomial such that  $N_i \sim \text{Binomial}(T, p_i)$  for each  $i=0,1,\dots,m$ .
4. The field reliability is binomially distributed with probability  $p$  of success.
5. Since the model does not use any temporal metrics, very weak tests can be combined with either the prior or the subsequent test and considered to be a single test. An example where this would be of value is if HASS vibration results in a TEF lower than 0.05, but it is determined that the HASS temperature test immediately following has failure events that might have been precipitated because of the combination of the sequence of the vibration and temperature environments. The new test stage would consist of both tests with  $N$  being the addition of the failure counts from each.
6. The TEF for the field environment is assumed to be equal to 1. This is in keeping with the intended use of a single-shot system in which the first field test is the final test of interest. Alternatively for continuous use systems, the TEF can be set at 1 at the termination of a warranty as long as no wear-out failures are anticipated.

## 2.4 Estimation Of Failure Counts At Test Stage

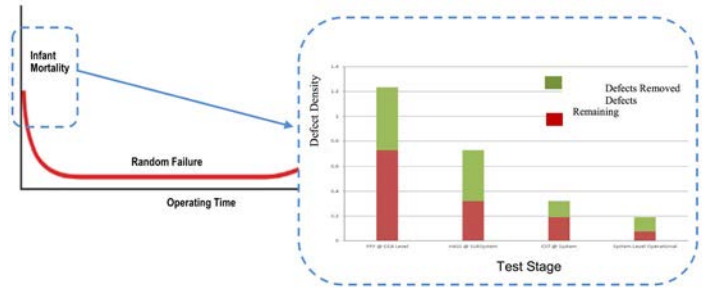
As described earlier, TEF adjustments can be used to normalize disparate tests to approximate constant stress. Under constant stress, the number of defects in a population during early screening has been shown to follow the basic exponential decay expression  $y = \alpha e^{-\beta t}$  by both theoretical and empirical methods (Defense 1993) (Fries and Sen 1996) (H.H. and C.P. 1992). At the conceptual level, this can be thought of as a rapid narrowing of the window of opportunity for a defect to survive one test but accumulate enough damage to fail the next test at the same stress level. This means that when the TEF adjustment is applied to failure count data from individual test phases of a screening program, the shape of the resultant defect density plot should approximate an exponential decay (figure 1). This is consistent with the models developed and employed by (Defense 1993) and (Fries and Sen 1996). The exponential decay of defects within a population and overall hazard rate during exposure to a constant stress in early life is similar to the infant mortality region of the bathtub curve (Modarres, Kaminskiy and Krivosv 2010) and is prevalent throughout reliability literature. A full justification for the use of an exponential decay shape is beyond the scope of this paper, but a thorough treatment can be found in (H.H. and C.P. 1992). While the cited literature is followed in this paper with the assumption of an exponential shape, circumstances may dictate that a different function be used with the described approach. In those cases, the discussion found later in this paper of the exponential parameters  $\alpha$  and  $\beta$  would not apply.

Ultimately the goal of any screening program is to ensure the items exiting the program are past the exponential decay portion of the “bathtub” curve and into the random failure portion. Using the TEF adjustment from equation (1), the failure count expression obtained is given by

$$N_i = T_i \cdot d_i \cdot \alpha \cdot e^{-\beta \cdot i} \quad (4)$$

From the expression found in Eq.4 one can also obtain the expression for the failure probability for an individual trial:

$$p_i = d_i \cdot \alpha \cdot e^{-\beta \cdot i} \quad (5)$$



**Figure 2** – Conceptual model of relationship between defects removed and defects remaining showing the exponential decay shape.

The equations found in (4) and (5) are nearly identical to the discrete exponential-growth model detailed in (Fries and Sen 1996). The only difference is the addition of the TEF adjustment term  $d_i$  into the equation to normalize the stress level. This constant-stress test performed multiple times is what Lloyd & Lipow were modeling in (Fries and Sen 1996) and is also the basis for most Discrete Reliability Growth Models (DRGM) (PM, WJ and WJ 2000).

The parameters found in (5) directly correlate to specific aspects of a testing effort. The  $d_i$  term represents the test effectiveness of the individual test stage  $i$ . The calculations of  $d_i$  for each test are done after a complete data set has been obtained. With the TEF adjustments applied, the other parameters within the model can be estimated. The  $\alpha$  parameter is related to the defect density of the particular lot of items being tested. The  $\beta$  parameter correlates to the overall effectiveness of the series of tests at screening out the defects and will be referred to as the stress constant of the screen.

## 2.5 Parameter Estimation And Field Reliability

As previously mentioned, empirical determination of the parameters requires a complete data set from the application of the screen and at least some results from field testing ( $m+1$  test stage). Once the TEF ( $d_i$ ) for each test stage is found, the  $\alpha$  and  $\beta$  can be estimated. The estimation procedure for  $\alpha$  and  $\beta$  after data collection is done by rearranging equation (4) to the following expression. After taking the natural log of each side, the expression takes on

$$\frac{N_i}{T_i} \cdot \frac{1}{d_i} = \alpha \cdot e^{-\beta \cdot i} \quad (6)$$

the form of the familiar linear equation of  $y = mx + b$ , where the left side of equation (7) is  $y$ ;  $\ln \alpha = b$ ;  $-\beta = m$ ; and  $i = x$ .

$$\ln\left(\frac{N_i}{T_i} \cdot \frac{1}{d_i}\right) = \ln \alpha - \beta \cdot i \quad (7)$$

With equation (4) in the form of (7), the failure count data of Ni and Ti at the individual stages of  $i=0, 1, \dots, m+1$  can be plotted. Then by applying a least-squares regression analysis,  $\alpha$  and  $\beta$  can be found. The resultant calculations for  $d_i$ ,  $\alpha$ , and  $\beta$  are useful on the lot that was just tested to determine the effectiveness of the entire screen, get an estimate on the initial defect density, and also make additional estimations on the fielded reliability as well as remaining defect density. Immediately following the screen, estimating the probability of success in the field from the screen data would be done by extending the expression in (5) to the  $i=m+1$  case and subtracting the result from 1. This expression can be found below.

$$R_{m+1} = 1 - \alpha \cdot e^{-\beta \cdot (m+1)} \quad (8)$$

Once calculations have been made for  $d_i$  and  $\beta$ , and since they are only dependent on the screen being applied, they can be held constant for the next lot of test items to go through the screen. This means that the only parameter left to be estimated is  $\alpha$  which is related to the defect density of the current lot. As a program matures, the reliability growth of that program will be reflected by a decreasing  $\alpha$  from lot to lot.

## 2.6 Model Applied to Synthetic Test Data

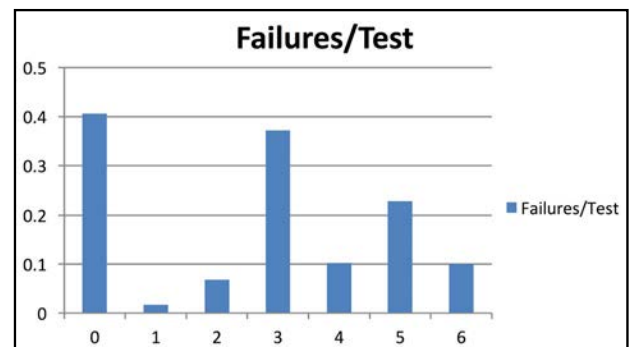
The data set in the following example is representative of data that comes from a screen applied to an electronic sub-assembly that is responsible for the control of a single-shot guided munition. The sub-assembly will be discussed in general terms and the failure data provided will be simulated but representative of data gathered. The example assumes that the second lot is entering the manufacturing phase before testing has been completed on the first lot. On a lot of 175 systems, all have been through the system functional test, but only 70 have had a field test, and only 10 have been used in the field.

The screening program that the control system goes through before being released for use is as follows; Stage 0) Full functional bench level testing at the card level, Stage 1) Full functional testing at the Sub-Assembly Level, Stage 2) HASS Vibration testing at the Sub-Assembly level, Stage

3) HASS Temperature testing at the Sub-Assembly level, Stage 4) Full Functional testing at the System Level, Stage 5) Field environment full functional system level test. The representative test data from this sequence of tests is seen in Table 1 and in graphical form as failures per test in Figure 3. The data shown in Table 1 and Figure 3 is reported at the Sub-Assembly level following the application of the apportionment method described in the “Data Required” section of this paper.

**Table 1** – Screening Program Results

Test Stage (i)	Test Description	Configuration	Failures (N <sub>i</sub> )	Total Tested (T <sub>i</sub> )
0	1st Functional	Card Level	71	175
1	F unctional	Sub-Assembly	3	175
2	HASS Vibe	Sub-Assembly	12	175
3	HASS Temp	Sub-Assembly	65	175
4	Functional	System	18	175
5	Field Test	System	16	70
6	Field	System	1	10



**Figure 3** – Unadjusted failures per test as a function of the test stage

Note that there is not a clear trend of decreasing failure rate in figure 3. This is due to the varying levels of stress applied. Because the approach described in this paper is not temporally based, there is flexibility in deciding the beginning or end of an individual test. In the data shown above, the first card level test is obviously important in precipitating infant mortality defects. The second test, first functional at the subassembly level, looks relatively weak. The second test is basically a repeat of multiple card level tests, but on a full subassembly and including verification of interconnect integrity between boards and chassis that could not be accomplished at the card level. The second

test can be combined with the first test to form a single test for scoring purposes. While this is not necessary for the model, it avoids an undesirable situation. With only three failures during test stage 1, the TEF will be low (0.019 in the example), meaning that the adjustment will be large. This makes this ineffective test very sensitive to every additional failure, such that a single failure will increase the failure rate 33%. The result will be such that even after the exponential decay has stabilized for the test program, this number will still be jumping around. This is obviously undesirable for tracking improvements.

A similar situation arises with the third and fourth test. The HASS vibration and HASS temperature tests are shown separately, with a low failure rate on the vibration test. Work has shown that failures precipitated by vibration will often not be detected until the assembly experiences rapid temperature swings, primarily because continuity may still exist across a broken solder joint if the two sides are in casual contact. This means that the failure rates of the two tests are interdependent and so can be combined into a single test for scoring purposes. The revised test results can be seen in Table 2.

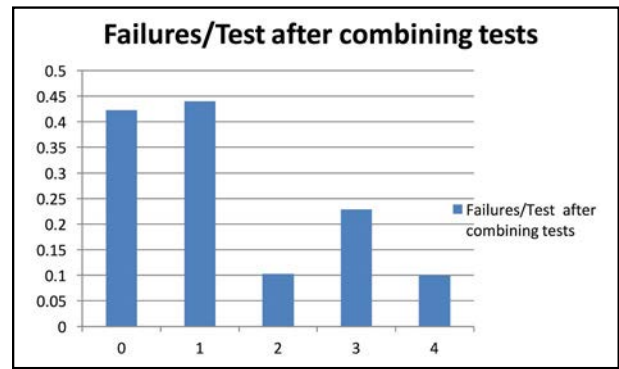
**Table 2** – Screening Program Revised Results

	Test Description	Failures (N <sub>i</sub> )	Total Tested (T <sub>i</sub> )
0	1st Functional	74	175
1	HASS	77	175
2	Functional	18	175
3	Field Test	16	70
0+1	Field	1	10

Note that there is still no clear trend of decreasing failure rate in figure 4.

## 2.7 Calculation And Application Of TEF Adjustment

In order to calculate the TEF adjustment of each test stage it is necessary to have a complete data set like that shown in Table 2. Then using expressions (1)-(3), the TEF at each stage can be calculated independently. An example calculation for test stage 1 from Table 2 can be seen below.



**Figure 4** – The revised plot of failure rate as a function of the test stage after combining ineffective tests with more effective tests that is sequentially adjacent.

$$\lambda_{1,1} = \frac{N_1}{T_1} = \frac{77}{175} = 0.440 \quad (9)$$

$$\lambda_{1,2} = \sum_{j=1}^4 \frac{N_j}{T_j} = \frac{77}{175} + \frac{18}{175} + \frac{16}{70} + \frac{1}{10} = 0.871 \quad (10)$$

$$d_1 = \frac{\lambda_{1,1}}{\lambda_{1,2}} = \frac{0.440}{0.871} = 0.505 \quad (11)$$

The TEF adjustments for stages  $i=0, 2$ , and  $3$  are easily calculated in the same manner as above. With the TEF adjustments calculated for each test stage one can begin to calculate the values on the left side of equation (6). These values represent the normalized failure rate of each test. These values can also be thought of as the defect density of each item prior to that test stage. In other words, if the test at stage  $i$  was 100% effective it would remove on average this amount of defects from each assembly. These values can be seen in Table 3.

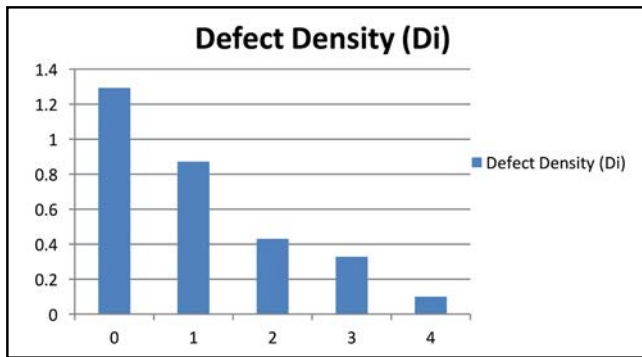
The graph in Figure 5 shows a plot of the defect density versus test stage after the TEF adjustment is applied. The figure illustrates the exponential decay of the overall defect density as a function of test stage. While the defects removed (failures/test as in Figure 4) at each test does not have a shape consistent with a parametric model, when one applies the TEF adjustments and plots the total defect density, a shape consistent with that of an exponential decay becomes apparent. The right side of the expression in (6) and the parameters  $\alpha$  and  $\beta$  that make up that expression are used to model this exponential decay

$$D_i = \frac{N_i}{T_i} \cdot \frac{1}{d_i}, \text{ left side of expression (6)}$$

Note that after the normalization based upon test strength, the defect density decay rate follows an exponential decay.

**Table 3** – Application of the TEFs result in estimation of defect density.

Test Stage (i)	TEF (d <sub>i</sub> )	Configuration	Defect Density (D <sub>i</sub> )
0	0.327	Card Level	1.294
1	0.505	Sub-Assembly	0.871
2	0.238	Sub-Assembly	0.431
3	0.696	Sub-Assembly	0.329
4	1	System	0.100



**Figure 5** – Plot of the defect density versus test stage after normalizing for test effectiveness.

## 2.8 Parameter Estimation Thru Least Squares Linear Regression

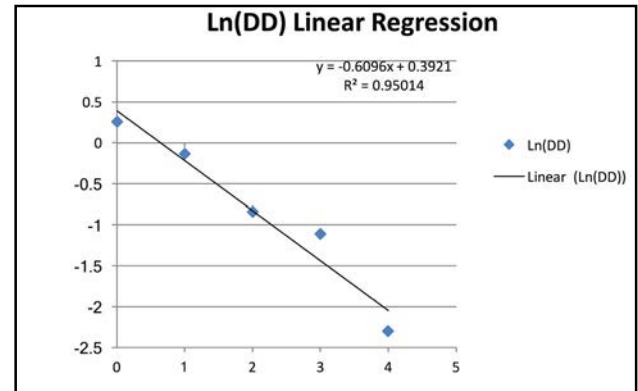
With the TEFs calculated and the resultant plot of defect density complete, one can now estimate the remaining parameters of  $\alpha$  and  $\beta$  with expressions (4)-(8). The simplest way to do this is by plotting the natural log of the defect density (left side of equation 7) against the test stage  $i$ . A least squares linear regression curve fit can then take place on the plot (Figure 6). Using the resultant linear equation one can solve for  $\alpha$  and  $\beta$ , where the relationship of  $\alpha$  and  $\beta$  to the straight line equation is described in expression (7). The calculations of  $\alpha$  and  $\beta$  can be seen below:

$$y = -0.6096x + 0.3921 \quad (12)$$

$$\beta = 0.6096 \quad (13)$$

$$\ln \alpha = 0.3921 \rightarrow \alpha = e^{0.3921} = 1.4 \quad (14)$$

With the  $\alpha$  and  $\beta$  calculated one could then make the point estimate for the probability of failure of an individual field event using expression 5.



**Figure 6** – Linear Regression of the Ln(Defect Density) versus test stage.

$$p_{m+1} = \alpha \cdot e^{-\beta \cdot (m+1)} = 1.48e^{-0.6096 \cdot 4} = 0.1292 \quad (15)$$

Note that while the field data showed 90% reliability in a sample of ten, the model shows that the actual field reliability will be:  $1 - 0.1292 = 0.871$  or 87.1% reliability.

## 3. Results & Discussion

With the model parameters that describe the test program estimated, when the next lot of items come into the testing program, one can start to analyze the results using the model to determine the likelihood that the lot will meet the field reliability requirement. The program can also set requirements at the early test stages that the program needs to achieve in order to realize the field reliability requirement. The first step in application of the model is to solve for the desired maximum failure rate at each test stage. These can be solved by rearranging expression 5 in order to solve for  $\alpha_{max}$  using the field requirement for  $p_i$ ,  $d_i=1$ ,  $\beta$ , and  $i=m+1$ . Included below is the example calculation for the data set of this paper with the field requirement set to 0.05 failure probability.

$$\alpha_{max} = \frac{p_{m+1}}{e^{-\beta \cdot (m+1)}} \quad (16)$$

$$\alpha_{max} = \frac{0.05}{e^{-0.6096 \cdot (4)}} = 0.5727 \quad (17)$$

Now that  $\alpha_{\max}$  is solved for, the maximum failure probabilities at each test stage can be calculated by using expression 5 and the TEFs. These values for the synthetic test data set are found in Table 4.

**Table 4** – Maximum allowable Failure Rates at each Test Stage

Test Stage (i)	Models Max allowable Pr(Failure)
0	0.1871
1	0.1572
2	0.0403
3	0.0639

When compared to the actual test results that this data was generated from these values are all lower than what was realized. This makes sense due to the fact that the model's actual projection (from the test data) for field probability of failure was 0.1292. So in order to get that projection down to 0.05 the requirement at each test stage is stiffer than what was experienced. With the limits set, when test results start to come in they can be used to calculate the probability that the requirement at that stage is going to be met when all of the items have been received. The details of this type of calculation are not covered in this paper, but are described in (Modarres, Kaminskiy and Krivosv 2010) and (O'Connor 2011). Also when test data comes in, calculations can be made towards solving for the  $\alpha$  of the current lot. Once an estimate for the  $\alpha$  of the current lot is made, a projection for field reliability can also be realized. This projection can provide a program with a useful tool early in a production lot to assess reliability.

## 4. Conclusion

This paper describes a novel approach for measuring and tracking reliability in single-shot systems with varying configuration and stress levels during test, and where no meaningful temporal metric such as MTBF can be applied. The most appropriate applications are programs that at some point have had a relatively stable hardware baseline over a statistically significant time interval, and an established test protocol. Under those circumstances, the approach can yield robust early predictions as to the reliability impacts of changes in manufacturer, manufacturing process, or

hardware. Similarly, when the manufacturing process and the hardware are stable, the approach can gauge the effect of changes in the test protocol.

While the emphasis here has been on single-shot systems, the use of a latent defect decay rate curve and test effectiveness factors has wider applicability to reusable systems that have a defined end-of-life requirement such as reaching the end of a warranty period with a desired reliability. The approach can be applied in any circumstances where the final testing phase can be assigned a TEF of "1."

## 5. Future Work

The primary limitation of the described model is the need for a full data set to determine the exponential decay parameters. This limitation can be mitigated somewhat by the use of Bayesian predicting based upon other systems that were tested in a similar manner. A Bayesian approach may be particularly well suited to situations where the test sequence changes during tracking. Also, estimation of TEF could be accomplished with a Monte Carlo approach.

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